SUPERSONIC FLOW OF A VISCOUS GAS IN THE REGION OF A WEAK SHOCK

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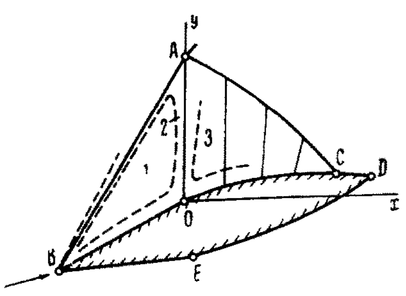
The flow of a viscous gas through a shock wave (strong shock) of thickness $O(\varepsilon^2)$, where $\varepsilon = R_0^{-1/2}$, and R_0 is the appropriate Reynolds number, has the character of a boundary layer flow and was investigated in [1]. A two-dimensional problem is discussed in the present paper, concerning the motion of the gas across a rectilinear acceleration discontinuity in an inviscid problem (a weak shock). It is shown that if the viscosity of the gas is taken into account in the neighborhood of the line of a weak shock, a 'boundary layer' of thickness $O(\varepsilon)$, is formed, in which the gas motion is described by a quasi-linear parabolic equation of the second order and unlike the shock wave, is essentially not of one-dimensional character. Also it is shown that on passing across the line of a weak shock, the terms $O(\varepsilon)$ in the gas parameters suffer a discontinuity (just as the tems $O(1)$ in the case of a shock wave), and formulas are found for these discontinuities.

1. Let us for example consider the problem of a profile $BOCDEB$ (see figure), with a wedge-like leading edge OBE, situated in a homogeneous, supersonic, viscous flow of a perfect gas. If the case of an inviscid flow is considered, then AB is a part of the bow shock wave the region AOB is the region of homogeneous flow, while the region AOC is the region of a simple wave. At the point O the curvature of the profile has a discontinuity and the straight line OA is a line of a weak shock. Let the Oy-axis of Cartesian coordinates coincide with the line OA; the parameters of the homogeneous flow in the region OAB will be denoted below by a subscript O, so that V_0 is the velocity and M_0 is the Mach No. in the region OAB; also



$$\frac{1}{V_0}u_0 = \frac{1}{M_0}, \qquad \frac{1}{V_0}v_0 = \frac{\sqrt{M_0^2-1}}{M_0}$$

Where u and v are the velocity components of the particles of the gas in the direction of x- and y-axes respectively. In the neighborhood of OA in the region of the simple wave we have

$$\frac{1}{V_0}u = \frac{1}{M_0} + \alpha\frac{x}{y - y_0} + O(x^2), \qquad \frac{1}{V_0}v = \frac{\sqrt{M_0^2 - 1}}{M_0} + O(x^2)$$
$$\left(\alpha = \frac{2}{\gamma + 1}\frac{\sqrt{M_0^2 - 1}}{M_0}\right) \tag{1.1}$$

where γ is the ratio of specific heats, and the constant y_0 is determined by the profile curvature at the point O when $x = +0$. If the profile curvature at $x = +0$ is zero (continuous), then $\alpha = 0$. In the case of a viscous gas, a boundary layer of thickness $O(\varepsilon)$, is formed near the surface of the profile, where $\varepsilon = R_0^{-1/2}$; the Reynolds No. R_0 is related to the parameters of the gas in the region OAB, the shock wave AB is changed into a region of thickness $O(\varepsilon^2)$, but, near OA, it forms a 'boundary layer', as will be shown later, of thickness $O(\varepsilon)$ (region 2 in the figure). The regions adjoining *2* are indicated in the figure by the numbers *1* and *3*. (We shall assume $l = OB$ to be a characteristic dimension). If the linear dimensions are related to l, the velocity components of the gas to V_0, the density ρ to ρ_0, the pressure p to $\rho_0 V_0^2$, the specific enthalpy i to V_0^2 , the coefficients of viscosity μ and λ to μ_0, then the equations of the laminar flow of a viscous gas (the Navier – Stokes equation) take the form

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{1}{R_0}\left\{2\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] + \frac{\partial}{\partial x}\left[\lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right]\right\}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \frac{1}{R_0}\left\{\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] + 2\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial y}\left[\lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right]\right\}$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \tag{1.2}$$

$$\frac{p}{\gamma - 1}\left[u\frac{\partial}{\partial x}\ln\frac{p}{\rho^\gamma} + v\frac{\partial}{\partial y}\ln\frac{p}{\rho^\gamma}\right] = \frac{1}{R_0}\left\{\frac{1}{\sigma}\left[\frac{\partial}{\partial x}\left(\mu\frac{\partial i}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial i}{\partial y}\right)\right] + \right.$$
$$\left. + \lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2 + 2\mu\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right\}$$

$$i = \frac{\gamma}{\gamma - 1}\frac{p}{\rho}, \quad \mu = \mu(i), \quad \lambda = \lambda(i), \quad R_0 = \frac{V_0\rho_0 l}{\mu_0}, \quad \sigma = \frac{c_p\mu}{k}$$

where R_0 is Reynolds number, σ is the Prandtl number, c_p is the specific heat at constant pressure, k is the coefficient of thermal conductivity for the gas. For the dimensionless values, the same nomenclature is observed as for the dimensional values.

2. In the regions *1* and *3* (see figure) the gas parameters can be represented in the form [2]

$$f = F_0(x, y) + \varepsilon F_1(x, y) + \varepsilon^2 F(x, y) + \ldots \qquad (\varepsilon = R_0^{-1/2}) \tag{2.1}$$

where u, v, p, ρ and i are understood to be represented by f. The terms $F_0(x, y)$ define the inviscid flow, the terms $F_1(x, y)$ indicate the influence of the displacement velocity of the boundary layer.

In the region *2* we will seek a solution to (1.2) in the form

$$f = f_0 + \varepsilon f_1(\xi, y) + \varepsilon^2 f_2(\xi, y) + \ldots \qquad (\xi = x\varepsilon^{-1})$$

i.e.

$$\begin{aligned}
u &= \frac{1}{M_0} + \varepsilon u_1(\xi, y) + \varepsilon^2 u_2(\xi, y) + \cdots \\
v &= \frac{\sqrt{M_0^2 - 1}}{M_0} + \varepsilon v_1(\xi, y) + \varepsilon^2 v_2(\xi, y) + \cdots \\
p &= \frac{1}{\gamma M_0^2} + \varepsilon p_1(\xi, y) + \varepsilon^2 p_2(\xi, y) + \cdots \\
\rho &= 1 + \varepsilon \rho_1(\xi, y) + \varepsilon^2 \rho_2(\xi, y) + \ldots \\
i &= \frac{1}{(\gamma - 1) M_0^2} + \varepsilon i_1(\xi, y) + \varepsilon^2 i_2(\xi, y) + \cdots
\end{aligned} \tag{2.2}$$

Rewriting the system of equations (1.2) in terms of the variables ξ and y, substituting (2.2) into it and equating the coefficients of like powers of ε, we obtain a system of equations defining the coefficients of the series of (2.2), which for the first two coefficients can be written in the form

$$\frac{\partial}{\partial \xi}(u_1 + M_0 p_1) = 0, \quad \frac{\partial v_1}{\partial \xi} = 0, \quad \frac{\partial}{\partial \xi}(\rho_1 + M_0 u_1) = 0, \quad \frac{\partial}{\partial \xi}(M_0^2 p_1 - \rho_1) = 0 \tag{2.3}$$

$$\frac{\partial}{\partial \xi}(u_2 + M_0 p_2) + (\rho_1 + M_0 u_1)\frac{\partial u_1}{\partial \xi} - M_0(\lambda_0 + 2\mu_0)\frac{\partial^2 u_1}{\partial \xi^2} + \sqrt{M_0^2 - 1}\,\frac{\partial u_1}{\partial y} = 0 \tag{2.4}$$

$$\frac{\partial v_2}{\partial \xi} + (\rho_1 + M_0 u_1)\frac{\partial v_1}{\partial \xi} - M_0 \mu_0 \frac{\partial^2 v_1}{\partial \xi^2} + \sqrt{M_0^2 - 1}\,\frac{\partial v_1}{\partial \xi} + M_0 \frac{\partial p_1}{\partial y} = 0 \tag{2.5}$$

$$\frac{\partial}{\partial \xi}(\rho_2 + M_0 u_2) + M_0 \frac{\partial}{\partial \xi}(\rho_1 u_1) + \frac{\partial}{\partial y}\left(\sqrt{M_0^2 - 1}\,\rho_1 + M_0 v_1\right) = 0 \tag{2.6}$$

$$\begin{aligned}
\frac{\partial}{\partial \xi}(M_0^2 p_2 - \rho_2) + M_0^3 u_1 \frac{\partial p_1}{\partial \xi} - (\gamma M_0^2 p_1 - \rho_1 + M_0 u_1)\frac{\partial \rho_1}{\partial \xi} + \\
+ \sqrt{M_0^2 - 1}\,\frac{\partial}{\partial y}(M_0^2 p_1 - \rho_1) - \frac{\gamma \mu_0 M_0^3}{\sigma}\frac{\partial^2}{\partial \xi^2}\left(p_1 - \frac{\rho_1}{\gamma M_0^2}\right) = 0
\end{aligned} \tag{2.7}$$

where λ_0 and μ_0 are the values of λ and μ for $i = 1 / (\gamma - 1)M_0^2$. From (2.3), it follows that

$$u_1 + M_0 p_1 = a_1(y), \quad v_1 = a_2(y), \quad \rho_1 + M_0 u_1 = a_3(y), \quad M_0 p_1 - \rho_1 = a_4(y) \tag{2.8}$$

Out of the four equations of (2.8), only three are independent. Substituting u_1 from the first equation into the third, we obtain $M_0^2 p_1 - \rho_1 = M_0 a_1(y) - a_3(y)$.) Fourth equation for finding u_1, v_1, p_1, and ρ_1 is obtained from the system of equations (2.4) to (2.8). Adding Equations (2.6) and (2.7), substracting Equation (2.4) from the result and multiplying subsequently by M_0, we obtain

$$\begin{aligned}
M_0 \frac{\partial}{\partial \xi}(\rho_1 u_1) + \frac{\partial}{\partial y}\left(\sqrt{M_0^2 - 1}\,\rho_1 + M_0 v_1\right) + M_0^3 u_1 \frac{\partial p_1}{\partial \xi} - (\gamma M_0^2 p_1 - \rho_1 + M_0 u_1)\frac{\partial \rho_1}{\partial \xi} + \\
+ \sqrt{M_0^2 - 1}\,\frac{\partial}{\partial y}(M_0^2 p_1 - \rho_1) - \frac{\gamma \mu_0 M_0^3}{\sigma}\frac{\partial^2}{\partial \xi^2}\left(p_1 - \frac{\rho_1}{\gamma M_0^2}\right) - (\rho_1 + M_0 u_1) M_0 \frac{\partial u_1}{\partial \xi} + \\
+ M_0^2(\lambda_0 + 2\mu_0)\frac{\partial^2 u_1}{\partial \xi^2} - M_0 \sqrt{M_0^2 - 1}\,\frac{\partial u_1}{\partial y} = 0
\end{aligned} \tag{2.9}$$

Equations (2.8) and (2.9) yield, after some transformations, an equation for u_1

$$M_0^2\left[\frac{(\gamma-1)\,\mu_0}{\sigma}+\lambda_0+2\mu_0\right]\frac{\partial^2 u_1}{\partial\xi^2}+[-(\gamma+1)\,M_0^2u_1+\gamma M_0^2a_1-a_3M_0]\,\frac{\partial u_1}{\partial\xi}- \\ -2M_0\sqrt{M_0^2-1}\,\frac{\partial u_1}{\partial y}+M_0\sqrt{M_0^2-1}\,\frac{da_1}{dy}+M_0\frac{dv_1}{dy}=0 \tag{2.10}$$

3. Equation (2.10) together with (2.8) describes asymptotically (as $\varepsilon\to 0$) the viscous gas flow near any rectilinear weak shock. We shall now digress from the problem of the profile and consider the case when the flow incident on the weak shock has the values u_1, v_1, p_1, and ρ_1 constant, supposing that the flow is one-dimensional. The equation (2.10) then takes the form

$$M_0^2\left[\frac{(\gamma-1)\,\mu_0}{\sigma}+\lambda_0+2\mu_0\right]\frac{d^2u_1}{d\xi^2}+[-(\gamma+1)\,M_0^2u_1+\gamma M_0^2a_1-a_3M_0]\,\frac{du_1}{d\xi}=0 \\ u_1\to u_1^-=\text{const}\quad \text{when}\ \ \xi\to-\infty \tag{3.1}$$

Integrating (3.1) with respect to ξ, we obtain

$$M_0^2\left[\frac{(\gamma-1)\,\mu_0}{\sigma}+\lambda_0+2\mu_0\right]\frac{du_1}{d\xi}=\frac{\gamma+1}{2}\,M_0^2u_1^2-(\gamma M_0^2a_1-a_3M_0)\,u_1+c\ \ (c=\text{const}) \tag{3.2}$$

Equation (3.2) can be presented in the form:

$$\left[\frac{(\gamma-1)\,\mu_0}{\sigma}+\lambda_0+2\mu_0\right]\frac{du_1}{d\xi}=\frac{\gamma+1}{2}\,(u_1-u_1^-)\,(u_1-u_1^+) \tag{3.3}$$

Where u_1^- and u_1^+ satisfy the relations

$$u_1^-+u_1^+=\frac{2}{(\gamma+1)\,M_0^2}\,(\gamma M_0^2a_1-a_3M_0),\qquad u_1^-u_1^+=\frac{2}{(\gamma+1)\,M_0^2}\,c \tag{3.4}$$

Since $u_1\to u_1^-$ when $\xi\to-\infty$, and u_1^- is a real number, therefore u_1^+ is a real number and $u_1\to u_1^+$ when $\xi\to+\infty$, and Equation (3.3) describes the gas flow in a weak shock wave. From (3.4), taking into account (2.8), the following formula is derived

$$u_1^+=\frac{\gamma-3}{\gamma+1}\,u_1^-+\frac{2\gamma}{\gamma+1}\,M_0\left(p_1^--\frac{\rho_1^-}{\gamma M_0^2}\right) \tag{3.5}$$

We should note that the value u_1^+ varies with the inclination of the weak shock (i.e. with M_0); for a direct shock $M_0=1$, and the formula (3.5) becomes a formula which can be obtained from the known results for a direct shock (see, for example, [3]).

4. From section 3 it follows that the flow near a weak shock has a substantially non-one-dimensional character; it is described by a parabolic quasi-linear equation (2.10). To find its solution in a concrete example the behavior of u_1 when $\xi\to\pm\infty$ must be known as well as the distribution of u_1 for certain values of y. In the problem with the profile, the behavior of u_1 with given y is defined by the solution of system (1 – 2) in the neighborhood of the point 0 (see figure). Let us determine the behavior of u_1 as $\xi\to\pm\infty$. In the regions *1* to *3* (see figure) u can be represented by a series of the type (2.1). For the region *1*

$$u=\frac{1}{M_0}+\varepsilon U_1^-(x,\,y)+O(\varepsilon^2)$$

while for the region *3* we have according to (1.1),

$$u=U_0^+(x,\,y)+\varepsilon U_1^+(x,\,y)+O(\varepsilon^2)=\frac{1}{M_0}+\frac{\alpha}{y-y_0}\,x+O(x^2)+\varepsilon U_1^+(x,\,y)+O(\varepsilon^2)$$

Assuming that $U_1^-(x,\,y)$, $U_1^+(x,\,y)$, . . . are represented asymptocially by a series in integral powers of x, we obtain

$$u = \frac{1}{M_0} + \varepsilon [U_1^-(0, y) + O(x)] + O(\varepsilon^2)$$
$$u = \frac{1}{M_0} + \frac{\alpha}{y - y_0} x + O(x^2) + \varepsilon [U_1^+(0, y) + O(x)] + O(\varepsilon^2) \tag{4.1}$$

Changing the variable to $\xi = x\varepsilon^{-1}$ and regrouping the terms of (4.1) result in the formulas

$$u = \frac{1}{M_0} + \varepsilon U_1^-(0, y) + O(\varepsilon^2) \qquad u = \frac{1}{M_0} + \varepsilon \left[\frac{\alpha}{y - y_0}\xi + U_1^+(0, y)\right] + O(\varepsilon^2) \tag{4.2}$$

The relations (4.2) are true for large ξ and small x. From (4.2) and (2.2) it follows that

$$u_1(\xi \to -\infty, y) = U_1^-(0, y) + \text{ the exponential terms}$$
$$u_1(\xi \to +\infty, y) = \frac{\alpha}{y - y_0}\xi + U_1^+(0, y) + \text{ the exponential terms} \tag{4.3}$$

The above relations are the conditions that the expansions (2.2) and (2.1) are not independent of each other.

5. We will now clarify whether Equation (2.10) has solutions with asymptotic behavior, described by the formulas (4.3), and how $U_1^-(0, y)$ and $U_1^+(0, y)$ are related. From equation (2.10) it follows that $u_1 \to U_1^-$ when $\xi \to -\infty$ ($v_1 \to V_1^-$, $p_1 \to P_1^-$), then

$$\frac{dU_1^-}{dy} - \frac{1}{2}\left(\frac{da_1}{dy} + \frac{1}{\sqrt{M_0^2 - 1}}\frac{dV_1^-}{dy}\right) = 0 \tag{5.1}$$

Substitution of a_1, according to (2.8) results, for $x = -0$, in

$$dU_1^- - \frac{dV_1^-}{\sqrt{M_0^2 - 1}} - M_0 dP_1^- = 0 \tag{5.2}$$

If the expansions (2.1) are substituted into the system of equations (1.2), in order to obtain a system of equations for the coefficients with the suffix 1, then (5.2) provides a relationship, which is fulfilled along the characteristic ($x = 0$) and will always be satisfied. After the integration of (5.1) with respect to y, we obtain, for $x = -0$,

$$U_1^- = \frac{1}{2}\left(a_1 + \frac{V_1^-}{\sqrt{M_0^2 - 1}} + c_0\right) \qquad (a_1 = U_1^- + M_0 P_1^-,\ c_0 = \text{const}) \tag{5.3}$$

Investigation of equation (2.10) shows that it has solutions with the following asymptotic behavior

$$\xi \to -\infty, \qquad u_1 = U_1^-(0, y) + g(y)e^{-\varphi(y)\xi^2} + \ldots$$

Where $g(y)$ is an arbitrary function of y

$$\varphi(y) = [c + \delta(y - y_0)]^{-1} > 0$$

(with the constant c suitably chosen)

$$\delta = \frac{2M_0}{\sqrt{M_0^2 - 1}}[(\gamma - 1)\mu_0\sigma^{-1} + \lambda_0 + 2\mu_0]$$

The decrease of the function $\varphi(y)$ with increasing y corresponds to the broadening of the region 2 (see the figure) with increasing distance from the profile.

For $\xi \to +\infty$ we shall seek an asymptotic representation, according to (4.3)

$$u_1 = \frac{\alpha}{y - y_0}\,\xi + \beta(y) + R(\xi, y) \qquad (\xi \to +\infty,\ R \to 0) \tag{5.4}$$

Substitution of (5.4) into (2.10) gives

$$\begin{aligned} &M_0^2[\sigma^{-1}(\gamma-1)\mu_0 + \lambda_0 + 2\mu_0]\frac{\partial^2 R}{\partial \xi^2} + \Big[-(\gamma+1)\Big(\frac{\alpha}{y-y_0}\,\xi + \beta + R\Big)M_0^2 + \gamma M_0^2 a_1 - \\ &- a_3 M_0\Big]\frac{\partial R}{\partial \xi} - (\gamma+1)M_0^2\frac{\alpha}{y-y_0}R - 2M_0\sqrt{M_0^2-1}\,\frac{\partial R}{\partial y} - M_0^2(\gamma+1)\frac{\alpha^2}{(y-y_0)^2}\,\xi - \\ &- M_0^2(\gamma+1)\frac{\alpha}{y-y_0}\,\beta + \frac{(\gamma M_0^2 a_1 - a_3 M_0)\,\alpha}{y-y_0} + 2M_0\sqrt{M_0^2-1}\,\frac{\alpha}{(y-y_0)^2}\,\xi - \\ &- 2M_0\sqrt{M_0^2-1}\,\frac{d\beta}{dy} + M_0\sqrt{M_0^2-1}\,\frac{da_1}{dy} + M_0\frac{dv_1}{dy} = 0 \end{aligned} \tag{5.5}$$

Equating the terms of like powers in (5.5) to zero, results in the following equations for α and β

$$\alpha\Big(\alpha - \frac{2}{\gamma+1}\,\frac{\sqrt{M_0^2-1}}{M_0}\Big) = 0 \tag{5.6}$$

The roots of (5.6) will be

$$\alpha_1 = \frac{2}{\gamma+1}\,\frac{\sqrt{M_0^2-1}}{M_0}$$

(satisfying exactly (1.1), and (4.3)), and $\alpha_2 = 0$. For the case $\alpha = \alpha_1$ the equation for β has the form

$$\frac{d\beta}{dy} + \frac{\beta}{y-y_0} = \frac{\gamma M_0 a_1 - a_3}{(\gamma+1)M_0}\,\frac{1}{y-y_0} + \frac{1}{2}\,\frac{d}{dy}\Big(a_1 + \frac{v_1}{\sqrt{M_0^2-1}}\Big) \tag{5.7}$$

For the case $\alpha = \alpha_2 = 0$ we have

$$\frac{d\beta}{dy} = \frac{1}{2}\,\frac{d}{dy}\Big(a_1 + \frac{v_1}{\sqrt{M_0^2-1}}\Big) \tag{5.8}$$

The general solution of (5.7) is given by the formula

$$\beta = \frac{1}{2}\Big(a_1 + \frac{v_1}{\sqrt{M_0^2-1}} + c_0\Big) + \frac{1}{y-y_0}\int\Big[\frac{\gamma M_0 a_1 - a_3}{(\gamma+1)M_0} - \frac{1}{2}\Big(a_1 + \frac{v_1}{\sqrt{M_0^2-1}} + c_0\Big)\Big]dy$$

which, by (2.8) and (5.3) together with the fact that $v_1 = a_2(y) = V_1^-$, becomes

$$\beta = U_1^- + \frac{1}{(\gamma+1)(y-y_0)}\int\Big[-2U_1^- + \gamma M_0\Big(P_1^- - \frac{\rho_1^-}{\gamma M_0^2}\Big)\Big]dy \tag{5.9}$$

Where ρ_1^- is the coefficient in the expansion (2.1), taken when $x = -0$. Inspection (5.5) for R shows that it possesses solutions with asymptotic properties, when $\xi \to +\infty$, of the form

$$R = G(y)e^{-\Phi(y)\xi^2} + \ldots, \qquad \Phi(y) = [c_1(y-y_0)^2 - \delta(y-y_0)]^{-1} > 0$$

where $G(y)$ is an arbitrary function of y, and constant c_1 is suitably chosen. From (4.3), (5.4) and (5.9) it follows, that

$$U_1^+ = U_1^- + [(\gamma+1)(y-y_0)]^{-1}\left\{\int_0^y\left[-2U_1^- + \gamma M_0\left(P_1^- - \frac{\rho_1^-}{\gamma M_0^2}\right)\right]dy + c\right\} \tag{5.10}$$

where c is a constant, U_1^+ is taken at $x = +0$, while the remaining functions at $x = -0$. The formula (5.10) shows that the terms O (ε) in the expansion (2.1) undergo a discontinuous change across the line of the weak shock ($x=0$), and at the same time a basic part of this discontinuity, which is defined by the integral in (5.10), depends on the dissipative processes in the region *2* (see the figure); constant c cannot be determined without consideration of the flow in the neighborhood of the point O. It is quite possible that the value of constant c is small in comparison with the integral, and it can be neglected, since c determines the influence of parts of the flow in the neighborhood of the point O, while the value of the integral depends on the acceleration in a simple wave when $x = +0$. For the case $\alpha = \alpha_2 = 0$ from (5.8) we obtain

$$\beta = U_1^+ = U_1^- + c \tag{5.11}$$

Where c is a constant, which can evidently be equal to zero. Thus, in the case where the profile curvature at the point O is continuous, the terms O (ε) in the transformations (2.1) are also continuous on the line of the weak shock.

6. Digressing from the problem of the profile, we will consider the case when the flow upstream of a line of the weak shock has constant terms O (ε) . From (5.10), following formula for this case

$$U_1^+ = U_1^- + \frac{1}{\gamma+1}\left[-2U_1^- + \gamma M_0\left(P_1^- - \frac{\rho_1^-}{\gamma M_0^2}\right)\right] + \frac{c_1}{(\gamma+1)(y-y_0)}, \qquad c_1 = \text{const}$$

which can be represented in the form

$$U_1^+ = \frac{\gamma-1}{\gamma+1}U_1^- + \frac{\gamma M_0}{\gamma+1}\left(P_1^- - \frac{\rho_1^-}{\gamma M_0^2}\right) + \frac{c_1}{(\gamma+1)(y-y_0)} \tag{6.1}$$

From (6.1) it follows, that if the term containing c_1 is neglected, U_1^+ is also constant, though different from u_1^+, given by the formula (3.5) for a weak shock. If it is additionally assumed that $P_1^- = p_1^- = \rho_1^- = 0$ in the formulas (3.5) and (6.1), then in the formula (3.5) the presence of u_1^- causes the appearance of u_1^+ with the opposite sign, while at the same time in (6.1), the sign of U_1^+ is the same as that of U_1^-.

7. In conclusion, we note that the character of the behavior of the terms O (ε) in expansions of the type (2.1), established for the case of a rectilinear shock, remains the same in the case of a curved weak shock (i.e. if the curvature of the surface of a body possesses a discontinuity at any point, then at the weak shock line originating from it, the tems O (ε) undergo discontinuities in expansions of the type (2.1); if the curvature is continuous, then the terms O (ε) are continuous).

BIBLIOGRAPHY

1. Germain P., Guirand, j.-P. Conditions de choc et structure des ondes de choc dans un écoulement stationnaire de fluide dissipatif. Publ. ONERA, No. 105, 1962.

2. Van Dyke M. Second-order compressible boundary layer theory with application to the

blunt bodies in hypersonic flow, Hypersonic Flow Research. Acad. Press. N.Y. – London, p. 37-76, 1962.

3. Kochin, N.E., Kibel', I.A. and Roze, N.V. Teoreticheskaia gidromekhanika (Theoretical Hydromechanics), Part 2, Gostekhizdat, pp. 378-382, 1948.

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